Model Persediaan Deterministik

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Notasi

C = purchase price per unit (if purchased) or the unit variable cost of production (if produced)

D = demand rate, units per year

A = fixed cost of a replenishment order (order cost) or setup cost of production

P = production or replenishment rate, units per year ($P > D$)

h = inventory carrying cost per unit per year (\$/unit/year), usually expressed as $h = iC$, where i is the annual inventory carrying cost rate

I_{max} = maximum on-hand inventory level, units

$\bar{I}$ = average on-hand inventory level, units

S_{max} = maximum shortage permitted, units

S = average shortage, units

r = reorder level, units

Q = order quantity, units

π = shortage cost per unit short, independent of the duration of the shortage

$\hat{\pi}$ = shortage cost per unit short per year

T = cycle length, the length of time between production runs

TC = total annual cost, which is a function of the inventory policy

l = lead time, the length of time between placement and receipt of orders

Model Produk Tunggal

In this model we consider an inventory system with a constant demand rate D. The production rate P is finite (that is, units produced are added to the inventory one at a time). The objectives of the analysis are to determine the optimal quantity Q^* to be ordered and the optimal shortage S^*_{max} allowed such that the total annual cost of the inventory system is minimized. Since the decision variables of this system are S^*_{max} and Q^*, the total-cost equation must be expressed in terms of S_{max} and Q. Other models will be presented later for which the total-cost equation is expressed in terms of other decision variables.

Ilustrasi Persediaan Deterministik Produk Tunggal

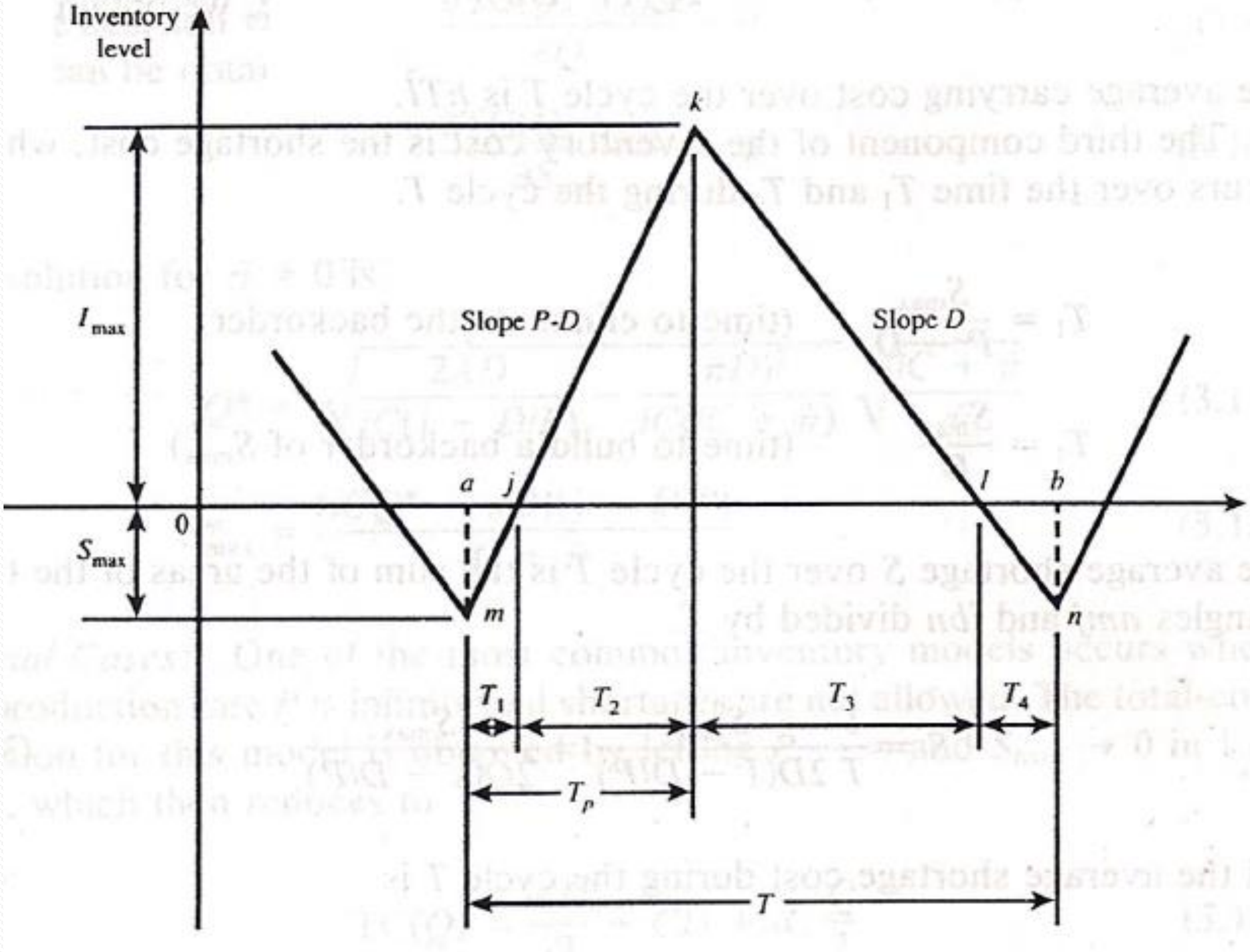

Figure . Single-product model with constant demand.

Deskripsi Sistem Persediaan Deterministik Produk Tunggal

The figure shows the change in the inventory level within a cycle T. When production starts (or orders start to arrive into the inventory) at point a, the inventory level will increase at a rate $P - D$, satisfying the backorders first and the current demand, until Q units are added to the inventory (point k). The inventory level will decrease at a rate D during the time period $T_3 + T_4$, and another cycle starts at point b, where the inventory level will increase at a rate $P - D$ until the quantity Q is produced, and so on.

Lot Production Time & Maximum Inventory Level

- Time to produce a lot Q

$$T_P = \frac{Q}{P} \quad (3.2)$$

- Maximum inventory level

$$I_{max} = \left[\frac{Q}{P}\right](P - D) - S_{max}$$

$$I_{max} = Q\left[1 - \frac{D}{P}\right] - S_{max} \quad (3.3)$$

Order Cost & Carrying Cost

The total-cost equation consists of several cost components. The first component is the cost of placing an order (or setup cost). This has already been expressed as the fixed amount A.

The second component of the average cycle cost is the cost of carrying the inventory for one cycle. This cost is incurred during time periods T_2 and T_3 (these are the only time periods for which we have inventory on hand during the total cycle T).

$$T_2 = \frac{I_{max}}{P - D}$$ (time to build the inventory from zero to I_{max})

$$T_3 = \frac{I_{max}}{D}$$ (time to consume the amount in inventory from I_{max} to zero)

Carrying Cost

The average inventory $\bar{I}$ over the cycle T is the area of the triangle *jkl* divided by T.

$$\bar{I} = \frac{1}{2T}(T_2 + T_3)I_{\max}$$

Substituting $T = Q/D$ and T_2, T_3, and $I_{\max}$ as given above, we obtain

$$\bar{I} = \frac{[Q(1 - D/P) - S_{\max}]^2}{2Q(1 - D/P)} \qquad (3.4)$$

The average carrying cost over the cycle T is $hT\bar{I}$.

Shortage Cost

The third component of the inventory cost is the shortage cost, which occurs over the time T_1 and T_4 during the cycle T.

$$T_1 = \frac{S_{\max}}{P - D} \qquad \text{(time to eliminate the backorder)}$$

$$T_4 = \frac{S_{\max}}{D} \qquad \text{(time to build a backorder of } S_{\max}\text{)}$$

The average shortage S over the cycle T is the sum of the areas of the two triangles amj and lbn divided by T.

$$S = \frac{1}{T}\frac{S_{\max}^2}{2D(1 - D/P)} = \frac{S_{\max}^2}{2Q(1 - D/P)} \qquad (3.5)$$

and the average shortage cost during the cycle T is

$$\hat{\pi}TS + \pi S_{\max}$$

Total Cost

The average cost per cycle is the sum of the ordering cost, item cost, carrying cost, and shortage cost:

$$A + CQ + hT\bar{I} + \hat{\pi}TS + \pi S_{\max} \tag{3.6}$$

The total annual cost is obtained by multiplying Eq. (3.6) by the number of orders per year, D/Q. By substituting $h = iC$, we obtain

$$\text{TC}(Q, S_{\max}) = \frac{AD}{Q} + CD + iC\bar{I} + \hat{\pi}S + \frac{\pi S_{\max} D}{Q} \tag{3.7}$$

Substitute for $\bar{I}$ and S from Eqs. (3.4) and (3.5):

$$\text{TC}(Q, S_{\max}) = \frac{AD}{Q} + CD + \frac{iC[Q(1 - D/P) - S_{\max}]^2}{2Q(1 - D/P)} + \frac{\hat{\pi}S_{\max}^2}{2Q(1 - D/P)} + \frac{\pi S_{\max} D}{Q} \tag{3.8}$$

Decision Variables

The decision variables of this model are Q and $S_{\max}$, and their optimum values can be obtained by solving the following simultaneous equations:

$$\frac{\partial \mathrm{TC}(Q, S_{\max})}{\partial Q} = 0 \tag{3.9}$$

$$\frac{\partial \mathrm{TC}(Q, S_{\max})}{\partial S_{\max}} = 0 \tag{3.10}$$

The solution for $\hat{\pi} \neq 0$ is

$$Q^* = \sqrt{\frac{2AD}{iC(1 - D/P)} - \frac{(\pi D)^2}{iC(iC + \hat{\pi})}} \sqrt{\frac{iC + \hat{\pi}}{\hat{\pi}}} \tag{3.11}$$

$$S_{\max}^* = \frac{(iCQ^* - \pi D)(1 - D/P)}{iC + \hat{\pi}} \tag{3.12}$$

Special Case 1

One of the most common inventory models occurs when the production rate P is infinite and shortages are not allowed. The total-cost equation for this model is obtained by letting $P \to \infty$ and $S_{max} \to 0$ in Eq. (3.8), which then reduces to

$$\mathrm{TC}(Q) = \frac{AD}{Q} + CD + iC\frac{Q}{2} \tag{3.13}$$

and the optimal quantity Q^* is given by

$$Q^* = \sqrt{\frac{2AD}{iC}} \tag{3.14}$$

Equation (3.14) is referred to as the EOQ (*economic order quantity*) model.

Special Case 2

If the shortage cost per unit short is zero (that is, $\pi = 0$), Eqs. (3.11) and (3.12) are reduced to

$$Q^*(\pi = 0) = \sqrt{\frac{2AD}{iC(1 - D/P)}} \sqrt{\frac{iC + \hat{\pi}}{\hat{\pi}}} \tag{3.15}$$

$$S^*_{\max}(\pi = 0) = \frac{iCQ^*(1 - D/P)}{iC + \hat{\pi}} \tag{3.16}$$

Substituting (3.15) and (3.16) into (3.8), respecting $\pi = 0$, yields

$$\mathrm{TC}^*(Q, S_{\max}) = CD + \sqrt{\frac{2ADiC(1 - D/P)\hat{\pi}}{iC + \hat{\pi}}} \tag{3.17}$$

Special Case 3

Another special case occurs when both π and $\hat{\pi}$ are finite while the production rate P is infinite. The equations corresponding to this case are obtained by taking the limit of Eqs. (3.8), (3.11), and (3.12) as $P \to \infty$. This yields

$$\mathrm{TC}(Q, S_{\max}) = \frac{AD}{Q} + CD + \frac{iC(Q - S_{\max})^2}{2Q} + \frac{S_{\max}(\hat{\pi} S_{\max} + 2\pi D)}{2Q} \tag{3.18}$$

$$Q^* = \sqrt{\frac{2AD}{iC} - \frac{(\pi D)^2}{iC(iC + \hat{\pi})}} \sqrt{\frac{iC + \hat{\pi}}{\hat{\pi}}} \tag{3.19}$$

$$S^*_{\max} = \frac{iCQ^* - \pi D}{iC + \hat{\pi}} \tag{3.20}$$

Special Case 4

The optimal value of Q^* for Eq. (3.15) is reduced to Eq. (3.21) when shortages are not allowed.

$$Q^* = \sqrt{\frac{2AD}{iC(1 - D/P)}} \tag{3.21}$$

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